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Deformation in micropolar cubic crystal due to various sources

Rajneesh Kumar [a], Praveen Ailawalia [b,*]

[a] Department of Mathematics, Kurukshetra University, Kurukshetra, Haryana, India
[b] Department of Applied Sciences, Institute of Engineering and Emerging Technologies, Makhnumajra, Baddi District Solan, HP, India

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Abstract

The response of a micropolar cubic crystal due to various sources has been studied. The eigenvalue approach using Laplace and Fourier transforms has been employed to solve the problem. The integral transforms have been inverted by using a numerical technique to obtain the displacement, microrotation and stress components in the physical domain. The results of normal displacement, normal force stress and tangential couple stress have been compared for micropolar cubic crystal and micropolar isotropic solid and illustrated graphically.

Keywords: Micropolar cubic crystal; Eigenvalue; Laplace and Fourier transforms

1. Introduction

The classical theory of elasticity does not explain certain discrepancies that occur in the case of problems involving elastic vibrations of high frequency and short wavelength, that is, vibrations due to the generation of ultrasonic waves. The reason lies in the microstructure of the material which exerts a special influence at high frequencies and short wavelengths.

An attempt was made to eliminate these discrepancies by suggesting that the transmission of interaction between two particles of a body through an elementary area lying within the material was affected not solely by the action of a force vector but also by a moment (couple) vector. This led to the existence of couple stress in elasticity. Polycrystalline materials, materials with fibrous or coarse grain structure come in this category. The analysis of such materials requires incorporating the theories of oriented media. For this reason, micropolar theories were developed by Eringen (1966a,b) for elastic solids and fluids.

* Corresponding author.
E-mail addresses: rajneesh_kuk@rediffmail.com (R. Kumar), praveen_2117@rediffmail.com (P. Ailawalia).

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Following various methods, the elastic fields of various loadings, inclusion and inhomogeneity problems, and interaction energy of point defects and dislocation arrangement have been discussed extensively in the past. Generally all materials have elastic anisotropic properties which mean the mechanical behavior of an engineering material is characterized by the direction dependence. However the three-dimensional study for an anisotropic material is much more complicated to obtain than the isotropic one, due to the large number of elastic constants involved in the calculation.

Because a wide class of crystals such as W, Si, Cu, Ni, Fe, Au, Al, etc., which are some frequent used substances, belong to cubic materials. The cubic materials have nine planes of symmetry whose normals are on the three coordinate axes and on the coordinate planes making an angle $\pi/4$ with the coordinate axes. With the chosen coordinate system along the crystalline directions, the mechanical behavior of a cubic crystal can be characterized by four independent elastic constants A_1, A_2, A_3 and A_4.

To understand the crystal elasticity of a cubic material, Chung and Buessem (1967) presented a convenient method to describe the degree of the elasticity anisotropy in a given cubic crystal. Later, Lie and Koehler (1968) used a Fourier expansion scheme to calculate the stress fields caused by a unit force in a cubic crystal. Steeds (1973) gave a complete discussion on the displacements, stresses and energy factors of the dislocations for two-dimensional anisotropic materials. Boulanger and Hayes (2000) investigated inhomogeneous plane waves in cubic elastic materials. Bertram et al. (2000) discussed generation of discrete isotropic orientation distributions for linear elastic cubic crystals. Kobayashi and Giga (2001) investigated anisotropy and curvature effects for growing crystals. Domanski and Jablonski (2001) studied resonances of nonlinear elastic waves in cubic crystal. Destrade (2001) considered the explicit secular equation for surface acoustic waves in monoclinic elastic crystals. Zhou and Ogawa (2002) investigated elastic solutions for a solid rotating disk with cubic anisotropy. Minagawa et al. (1981) discussed the propagation of plane harmonic waves in a cubic micropolar medium. Recently Kumar and Rani (2003) studied time harmonic sources in a thermally conducting cubic crystal. However no attempt has been made to study source problems in micropolar cubic crystals.

The present investigation is to determine the components of displacement, microrotation and stresses in micropolar cubic crystal due to concentrated force, uniformly distributed force and linearly distributed force. The solutions are obtained by using eigenvalue approach after employing integral transformation technique. The integral transforms are inverted using a numerical method.

2. Problem formulation

We consider a homogeneous micropolar cubic crystal of infinite extent with Cartesian coordinate system (x, y, z). To analyze the displacements, microrotation and stresses at the interior of the medium due to various sources, the continuum is divided into two half-spaces defined by

(i) half space I $|x| < \infty$, $-\infty < y \leqslant 0$, $|z| < \infty$,
(ii) half space II $|x| < \infty$, $0 \leqslant y < \infty$, $|z| < \infty$.

If we restrict our analysis to the plane strain parallel to xy-plane with displacement vector $\vec{u} = (u_1, u_2, 0)$ and microrotation vector $\vec{\phi} = (0, 0, \phi_3)$ then the field equations and constitutive relations for such a medium in the absence of body forces and body couples given by Minagawa et al. (1981) can be recalled as

$$A_1 \frac{\partial^2 u_1}{\partial x^2} + A_3 \frac{\partial^2 u_1}{\partial y^2} + (A_2 + A_4) \frac{\partial^2 u_2}{\partial x \partial y} + (A_3 - A_4) \frac{\partial \phi_3}{\partial y} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{1}$$

$$A_3 \frac{\partial^2 u_2}{\partial x^2} + A_1 \frac{\partial^2 u_2}{\partial y^2} + (A_2 + A_4) \frac{\partial^2 u_1}{\partial x \partial y} - (A_3 - A_4) \frac{\partial \phi_3}{\partial x} = \rho \frac{\partial^2 u_2}{\partial t^2}, \tag{2}$$

$$B_3\nabla^2\phi_3 + (A_3 - A_4)\left(\frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y}\right) - 2(A_3 - A_4)\phi_3 = \rho j\frac{\partial^2\phi_3}{\partial t^2}, \tag{3}$$

$$t_{22} = A_2\frac{\partial u_1}{\partial x} + A_1\frac{\partial u_2}{\partial y}, \tag{4}$$

$$t_{21} = A_4\left(\frac{\partial u_2}{\partial x} - \phi_3\right) + A_3\left(\frac{\partial u_1}{\partial y} + \phi_3\right), \tag{5}$$

$$m_{23} = B_3\frac{\partial\phi_3}{\partial y}, \tag{6}$$

where t_{22}, t_{21}, m_{23} are the components of normal force stress, tangential force stress and tangential couple stress respectively A_1, A_2, A_3, A_4, B_3 are characteristic constants of the material, ρ is the density and j is the microinertia and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

Introducing the dimensionless variables defined by the expressions

$$\begin{aligned} &x' = \frac{\omega^*}{c_1}x, \quad y' = \frac{\omega^*}{c_1}y, \quad u_1' = \frac{\omega^*}{c_1}u_1, \quad u_2' = \frac{\omega^*}{c_1}u_2, \quad \phi_3' = \frac{A_1}{A_4}\phi_3, \\ &\{t_{22}', t_{21}'\} = \frac{(t_{22}, t_{21})}{A_1}, \quad m_{23}' = \frac{c_1}{B_3\omega^*}m_{23}, \quad t' = \omega^* t, \quad a' = \frac{\omega^*}{c_1}a, \end{aligned} \tag{7}$$

where

$$\omega^{*2} = \frac{A_4 - A_3}{\rho j}, \quad c_1^2 = \frac{A_1}{\rho}. \tag{8}$$

Using (7), the system of Eqs. (1)–(3) reduce to (dropping the primes),

$$A_1\frac{\partial^2 u_1}{\partial x^2} + A_3\frac{\partial^2 u_1}{\partial y^2} + (A_2 + A_4)\frac{\partial^2 u_2}{\partial x\,\partial y} + \frac{A_4(A_3 - A_4)}{A_1}\frac{\partial\phi_3}{\partial y} = \rho c_1^2\frac{\partial^2 u_1}{\partial t^2}, \tag{9}$$

$$A_3\frac{\partial^2 u_2}{\partial x^2} + A_1\frac{\partial^2 u_2}{\partial y^2} + (A_2 + A_4)\frac{\partial^2 u_1}{\partial x\,\partial y} - \frac{A_4(A_3 - A_4)}{A_1}\frac{\partial\phi_3}{\partial x} = \rho c_1^2\frac{\partial^2 u_2}{\partial t^2}, \tag{10}$$

$$B_3\frac{A_4\omega^{*2}}{A_1c_1^2}\nabla^2\phi_3 + (A_3 - A_4)\left(\frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y}\right) - 2\frac{A_4(A_3 - A_4)}{A_1}\phi_3 = \rho j\omega^{*2}\frac{A_4}{A_1}\frac{\partial^2\phi_3}{\partial t^2}. \tag{11}$$

The initial conditions are given by

$$\begin{aligned} &u_n(x, y, 0) = \dot{u}_n(x, y, 0) = 0; \quad n = 1, 2, \\ &\phi_3(x, y, 0) = \dot{\phi}_3(x, y, 0) = 0. \end{aligned} \tag{12}$$

Applying the Laplace transform with respect to time 't' defined by

$$\{\bar{u}_n(x, y, p), \bar{\phi}_3(x, y, p)\} = \int_0^\infty \{u_n(x, y, t), \phi_3(x, y, t)\}\mathrm{e}^{-pt}\,\mathrm{d}t, \quad n = 1, 2 \tag{13}$$

and then the Fourier transform with respect to 'x' defined by

$$\{\tilde{u}_n(\xi, y, p), \tilde{\phi}_3(\xi, y, p)\} = \int_{-\infty}^\infty \{\bar{u}_n(x, y, p), \bar{\phi}_3(x, y, p)\}\mathrm{e}^{\mathrm{i}\xi x}\,\mathrm{d}x, \quad n = 1, 2 \tag{14}$$

on Eqs. (9)–(11) and with the help of initial conditions (12), we obtain

$$D^2\tilde{u}_1 = b_{11}\tilde{u}_1 + a_{12}D\tilde{u}_2 + a_{13}D\tilde{\phi}_3, \tag{15}$$

$$D^2\tilde{u}_2 = b_{22}\tilde{u}_2 + a_{21}D\tilde{u}_1 + b_{23}\tilde{\phi}_3, \tag{16}$$

$$D^2\tilde{\phi}_3 = b_{33}\tilde{\phi}_3 + a_{31}D\tilde{u}_1 + b_{32}D\tilde{u}_2, \tag{17}$$

where

$$\begin{aligned}
&b_{11} = \frac{\rho c_1^2 p^2 + \xi^2 A_1}{A_3}, \quad b_{22} = \frac{\rho c_1^2 p^2 + \xi^2 A_3}{A_1}, \quad b_{23} = -\frac{\mathrm{i}\xi A_4(A_3 - A_4)}{A_1^2}, \\
&b_{32} = \frac{\mathrm{i}\xi c_1^2 A_1(A_3 - A_4)}{\omega^{*2} A_4 B_3}, \quad b_{33} = \frac{1}{B_3}\left[\xi^2 B_3 + \rho j c_1^2 p^2 + 2(A_3 - A_4)\frac{c_1^2}{\omega^{*2}}\right], \\
&a_{13} = -\frac{A_4(A_3 - A_4)}{A_1 A_3}, \quad a_{21} = \frac{\mathrm{i}\xi(A_2 + A_4)}{A_1}, \quad a_{31} = \frac{A_1(A_3 - A_4)c_1^2}{A_4 B_3 \omega^{*2}}, \\
&a_{12} = \frac{\mathrm{i}\xi(A_2 + A_4)}{A_3}, \quad D = \frac{\mathrm{d}}{\mathrm{d}y}.
\end{aligned} \tag{18}$$

Eqs. (15)–(17) may be written as

$$DW(\xi, y, p) = A(\xi, p)W(\xi, y, p), \tag{19}$$

where

$$\begin{aligned}
&W = \begin{pmatrix} V \\ DV \end{pmatrix}, \quad A = \begin{pmatrix} O & I \\ A_1^* & A_2^* \end{pmatrix}, \quad V = \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{\phi}_3 \end{pmatrix}, \\
&A_1^* = \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \end{pmatrix}, \quad A_2^* = \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \end{pmatrix},
\end{aligned} \tag{20}$$

O and I are respectively zero and identity matrix of order 3.

To solve Eq. (19), we assume

$$W(\xi, y, p) = X(\xi, p)\mathrm{e}^{qy}, \tag{21}$$

which leads to eigenvalue problem. The characteristic equation corresponding to matrix A is given by

$$|A - qI| = 0, \tag{22}$$

which on expansion provides us

$$q^6 + \lambda_1 q^4 + \lambda_2 q^2 + \lambda_3 = 0, \tag{23}$$

where

$$\begin{aligned}
&\lambda_1 = -(a_{12}a_{21} + a_{13}a_{31} + b_{11} + b_{22} + b_{33}), \\
&\lambda_2 = a_{12}(a_{21}b_{33} - b_{23}a_{31}) + a_{13}(b_{22}a_{31} - a_{21}b_{32}) + b_{22}b_{33} - b_{23}b_{32} + b_{11}(b_{22} + b_{33}), \\
&\lambda_3 = b_{11}(b_{23}b_{32} - b_{22}b_{33}).
\end{aligned} \tag{24}$$

The eigenvalues of the matrix A are the characteristic roots of Eq. (23). The vectors $X(\xi,p)$ corresponding to the eigenvalues q_s can be determined by solving the homogeneous equation

$$[A - qI]X(\xi, p) = 0. \tag{25}$$

The set of eigenvectors $X_s(\xi,p),\ s=1,2,\ldots,6$ may be obtained as

$$X_s(\xi,p)=\begin{pmatrix}X_{g1}(\xi,p)\\X_{g2}(\xi,p)\end{pmatrix}, \tag{26}$$

where

$$X_{g1}(\xi,p)=\begin{pmatrix}q_g\\a_g\\b_g\end{pmatrix},\qquad X_{g2}(\xi,p)=\begin{pmatrix}q_g^2\\a_gq_g\\b_gq_g\end{pmatrix},\quad q=q_g;\ \ g=1,2,3, \tag{27}$$

$$X_{R1}(\xi,p)=\begin{pmatrix}-q_R\\a_R\\b_R\end{pmatrix},\qquad X_{R2}(\xi,p)=\begin{pmatrix}q_R^2\\-a_Rq_R\\-b_Rq_R\end{pmatrix},\quad R=g+3;\ \ q=-q_g;\ \ g=1,2,3 \tag{28}$$

and

$$\begin{aligned}a_g&=\frac{b_{11}b_{23}-q_g^2(b_{23}+a_{21}a_{13})}{\nabla_g},\\ b_g&=\frac{q_g^2a_{31}+a_gb_{32}}{q_g^2-b_{33}},\\ \nabla_g&=q_g^2a_{13}+a_{12}b_{23}-b_{22}a_{13}.\end{aligned} \tag{29}$$

The solution of Eq. (21) is given by

$$W(\xi,y,p)=\sum_{s=1}^{3}[B_sX_s(\xi,p)\exp(q_sy)+B_{s+3}X_{s+3}(\xi,p)\exp(-q_sy)], \tag{30}$$

where $B_\Xi(\Xi=1,2,\ldots,6)$ are arbitrary constants.

Eq. (30) represents the solution of the general problem in case of micropolar cubic crystals and can be applied to a class of problem in the domain of Laplace and Fourier transforms.

3. Application

3.1. Mechanical sources

We consider an infinite micropolar cubic crystal in which a normal force $F_0=F\psi(x)\delta(t)$ is acting at the origin of the Cartesian coordinate system, where $\delta(t)$ is Dirac delta function, F is the magnitude of force applied and $\psi(x)$ specify the vertical load distributed function along x-axis. Also the components of displacement, microrotation, tangential force stress and tangential couple stress must be continuous across the interface. Mathematically the boundary conditions at the interface of two half-spaces $y=0$ are given by

$$\begin{aligned}&u_1(x,0^+,t)-u_1(x,0^-,t)=0,\quad u_2(x,0^+,t)-u_2(x,0^-,t)=0,\quad \phi_3(x,0^+,t)-\phi_3(x,0^-,t)=0,\\ &t_{22}(x,0^+,t)-t_{22}(x,0^-t)=-F\psi(x)\delta(t),\quad t_{21}(x,0^+,t)-t_{21}(x,0^-,t)=0,\\ &m_{23}(x,0^+,t)-m_{23}(x,0^-,t)=0.\end{aligned} \tag{31}$$

Using (7) and applying Laplace and Fourier transforms defined by (13) and (14) on Eq. (31), after suppressing the primes, and with the help of Eq. (30), we obtain the transformed components of displacement, microrotation and stresses as

$$\tilde{u}_1 = \frac{-1}{\Delta}[q_1\Delta e^{-q_1y} + q_2\Delta_2 e^{-q_2y} + q_3\Delta_3 e^{-q_3y} - q_1\Delta_4 e^{q_1y} - q_2\Delta_5 e^{q_2y} - q_3\Delta_6 e^{q_3y}], \tag{32}$$

$$\tilde{u}_2 = \frac{1}{\Delta}[a_1\Delta_1 e^{-q_1y} + a_2\Delta_2 e^{-q_2y} + a_3\Delta_3 e^{-q_3y} + a_1\Delta_4 e^{q_1y} + a_2\Delta_5 e^{q_2y} + a_3\Delta_6 e^{q_3y}], \tag{33}$$

$$\tilde{\phi}_3 = \frac{1}{\Delta}[b_1\Delta_1 e^{-q_1y} + b_2\Delta_2 e^{-q_2y} + b_3\Delta_3 e^{-q_3y} + b_1\Delta_4 e^{q_1y} + b_2\Delta_5 e^{q_2y} + b_3\Delta_6 e^{q_3y}], \tag{34}$$

$$\tilde{t}_{22} = \frac{1}{\Delta}[r_1\Delta_1 e^{-q_1y} + r_2\Delta_2 e^{-q_2y} + r_3\Delta_3 e^{q_3y} - r_1\Delta_4 e^{q_1y} - r_2\Delta_5 e^{q_2y} - r_3\Delta_6 e^{q_3y}], \tag{35}$$

$$\tilde{t}_{21} = \frac{1}{\Delta}[s_1\Delta_1 e^{-q_1y} + s_2\Delta_2 e^{-q_2y} + s_3\Delta_3 e^{-q_3y} - s_1\Delta_4 e^{q_1y} - s_2\Delta_5 e^{q_2y} - s_3\Delta_6 e^{q_3y}], \tag{36}$$

$$\tilde{m}_{23} = -\frac{K}{A_{11}\Delta}[b_1q_1\Delta_1 e^{-q_1y} + b_2q_2\Delta_2 e^{-q_2y} + b_3q_3\Delta_3 e^{-q_3y} - b_1q_1\Delta_4 e^{q_1y} - b_2q_2\Delta_5 e^{q_2y} - b_3q_3\Delta_6 e^{q_3y}], \tag{37}$$

where

$$\begin{aligned}
&\Delta_{1,4} = \pm 4F\tilde{\psi}(\xi)h_1G, \quad \Delta_{2,5} = \mp 4F\tilde{\psi}(\xi)h_2G, \quad \Delta_{3,6} = \pm 4F\tilde{\psi}(\xi)h_3G,\\
&\Delta = 8G[s_1(a_2b_3 - a_3b_2) - s_2(a_1b_3 - a_3b_1) + s_3(a_1b_2 - a_2b_1)],\\
&G = r_1q_2q_3(b_3 - b_2) - r_2q_1q_3(b_3 - b_1) + r_3q_1q_2(b_2 - b_1),\\
&h_{1,2,3} = s_{3,3,2}a_{2,1,1} - s_{2,1,1}a_{3,3,2}, \quad s_g = \frac{1}{A_1}\left[-i\xi a_g A_4 + A_3q_g^2 + b_g(A_3 - A_4)\frac{A_4}{A_1}\right],\\
&r_g = q_g\left[i\xi\frac{A_2}{A_1} - a_g\right], \quad g = 1,2,3.
\end{aligned} \tag{38}$$

3.1.1. Concentrated normal force

In order to determine displacements, microrotation and stresses due to concentrated force described as Dirac delta function, $\psi(x) = \delta(x)$ must be used. The Fourier transform of $\psi(x)$ with respect to pair (x, ξ) will be $\tilde{\psi}(\xi) = 1$.

3.1.2. Uniformly distributed force

The solution due to uniformly distributed force is obtained by setting

$$\psi(x) = \begin{bmatrix} 1 & \text{if } |x| \leqslant a, \\ 0 & \text{if } |x| > a, \end{bmatrix}$$

in Eq. (31). The Fourier transform with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width $2a$ applied at the origin of the coordinate system ($x = y = 0$) in dimensionless form after suppressing the primes becomes

$$\tilde{\psi}(\xi) = \left[2\sin\left(\frac{\xi c_1 a}{\omega^*}\right)\Big/\xi\right], \quad \xi \neq 0. \tag{39}$$

3.1.3. Linearly distributed force

The solution due to linearly distributed force is obtained by substituting

$$\psi(x) = \begin{bmatrix} 1 - \dfrac{|x|}{a} & \text{if } |x| \leqslant a, \\ 0 & \text{if } |x| > a, \end{bmatrix} \tag{40}$$

in Eq. (31). The Fourier transform of $\psi(x)$ in dimensionless form after suppressing the primes is

$$\tilde{\psi}(\xi) = \frac{2\left[1 - \mathrm{Cos}\left(\frac{\xi c_1 a}{\omega^*}\right)\right]}{\frac{\xi^2 c_1 a}{\omega^*}}. \tag{41}$$

The expressions for the components of displacement, microrotation, force stress and couple stress may be obtained as in Eqs. (32)–(37), by replacing $\widetilde{\psi}(\xi)$ by 1, $\left[2\sin\left(\frac{\xi c_1 a}{\omega^*}\right)/\xi\right]$ and $\frac{2\left[1-\mathrm{Cos}\left(\frac{\xi c_1 a}{\omega^*}\right)\right]}{\frac{\xi^2 c_1 a}{\omega^*}}$ in case of concentrated force, uniformly distributed force and linearly distributed force respectively.

3.2. Particular case

Taking $A_1 = \lambda + 2\mu + K$, $A_2 = \lambda$, $A_3 = \mu + K$, $A_4 = \mu$, $B_3 = \gamma$, in Eqs. (32)–(37) with (39)–(41) we obtain the corresponding expressions in micropolar isotropic medium for concentrated force, uniformly distributed force and linearly distributed force respectively. These results tally with the one if we solve the problem in micropolar isotropic medium.

4. Inversion of the transform

The transformed displacements and stresses are functions of y, the parameters of Laplace and Fourier transforms p and ξ respectively, and hence are of the form $\tilde{f}(\xi, y, p)$. To get the function in the physical domain, first we invert the Fourier transform using

$$\bar{f}(x,y,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\xi x}\tilde{f}(\xi,y,p)\,\mathrm{d}\xi = \frac{1}{\pi}\int_0^{\infty}\{\cos(\xi x)f_\mathrm{e} - \mathrm{i}\sin(\xi x)f_\mathrm{o}\}\,\mathrm{d}\xi, \tag{42}$$

where f_e and f_o are even and odd parts of the function $\tilde{f}(\xi, y, p)$ respectively. Thus, expressions (42) give us the transform $\bar{f}(x, y, p)$ of the function $f(x, y, t)$.

Now, for the fixed values of ξ, x and y, the $\bar{f}(x, y, p)$ in the expression (40) can be considered as the Laplace transform $\bar{g}(p)$ of some function $g(t)$. Following Honig and Hirdes (1984), the Laplace transformed function $\bar{g}(p)$ can be converted as given below.

The function $g(t)$ can be obtained by using

$$g(t) = \frac{1}{2\pi\mathrm{i}}\int_{C-\mathrm{i}\infty}^{C+\mathrm{i}\infty} \mathrm{e}^{pt}\bar{g}(p)\,\mathrm{d}p, \tag{43}$$

where C is an arbitrary real number greater than all the real parts of the singularities of $\bar{g}(p)$. Taking $p = C + \mathrm{i}y$, we get

$$g(t) = \frac{\mathrm{e}^{Ct}}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}ty}\bar{g}(C + \mathrm{i}y)\,\mathrm{d}y. \tag{44}$$

Now, taking $\mathrm{e}^{-Ct}g(t)$ as $h(t)$ and expanding it as Fourier series in $[0, 2L]$ we obtain approximately the formula

$$g(t) = g_\infty(t) + E_{D'},$$

where

$$\begin{aligned} g_\infty(t) &= \frac{C_0}{2} + \sum_{k=1}^{\infty} C_k, \quad 0 \leqslant t \leqslant 2L, \\ C_k &= \frac{\mathrm{e}^{Ct}}{L}\Re\left[\mathrm{e}^{\frac{\mathrm{i}k\pi t}{L}}\bar{g}\left(C + \frac{\mathrm{i}k\pi}{L}\right)\right], \end{aligned} \tag{45}$$

E_D is the discretization error and can be made arbitrary small by choosing C large enough. The value of C and L are chosen according to the criteria outlined by Honig and Hirdes (1984).

Since the infinite series in Eq. (45) can be summed up only to a finite number of N terms, so the approximate value of $g(t)$ becomes

$$g_N(t) = \frac{C_0}{2} + \sum_{k=1}^{N} C_k, \quad 0 \leqslant t \leqslant 2L. \tag{46}$$

Now, we introduce a truncation error E_T that must be added to the discretization error to produce the total approximation error in evaluating $g(t)$ using the above formula. Two methods are used to reduce the total error. The discretization error is reduced by using the "Korrecktur"-method, Honig and Hirdes (1984) and then "ε-algorithm" is used to reduce the truncation error and hence to accelerate the convergence.

The " Korrecktur"-method formula, to evaluate the function $g(t)$ is

$$g(t) = g_\infty(t) - e^{-2CL} g_\infty(2L + t) + E_{D'}, \tag{47}$$

where

$$|E_{D'}| \ll |E_D|\cdot \tag{48}$$

Thus, the approximate value of $g(t)$ becomes

$$g_{N_k}(t) = g_N(t) - e^{-2CL} g_{N'}(2L + t), \tag{49}$$

where, N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm which is used to accelerate the convergence of the series in Eq. (46). Let N be a natural number and $S_m = \sum_{k=1}^{m} C_k$ be the sequence of partial sums of Eq. (46). We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \quad \varepsilon_{1,m} = S_m,$$

$$\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n,m}}; \quad n, m = 1, 2, 3 \ldots$$

It can be shown Honig and Hirdes (1984) that the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \ldots, \varepsilon_{N,1}$ converge to $g(t) + E_D - C_0/2$ faster than the sequence of partial S_m,$m = 1, 2, 3, \ldots$ The actual procedure to invert the Laplace transform reduces to the study of Eq. (47) together with the ε-algorithm.

The last step is to evaluate the integral in Eq. (42). The method for evaluating this integral is given by Press et al. (1986) and which involves the use of Rhomberg's integration with adaptive step size. This also uses the results from successive refinement of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

5. Numerical results and discussions

For numerical computations, we take the following values of relevant parameters for micropolar cubic crystal as

$$A_1 = 13.97 \times 10^{10}\ \text{dyne/cm}^2, \quad A_3 = 3.2 \times 10^{10}\ \text{dyne/cm}^2, \quad A_2 = 13.75 \times 10^{10}\ \text{dyne/cm}^2,$$
$$A_4 = 2.2 \times 10^{10}\ \text{dyne/cm}^2, \quad B_3 = 0.056 \times 10^{10}\ \text{dynes}.$$

For the comparison with micropolar isotropic solid, following Gauthier (1982), we take the following values of relevant parameters for the case of aluminium epoxy composite as

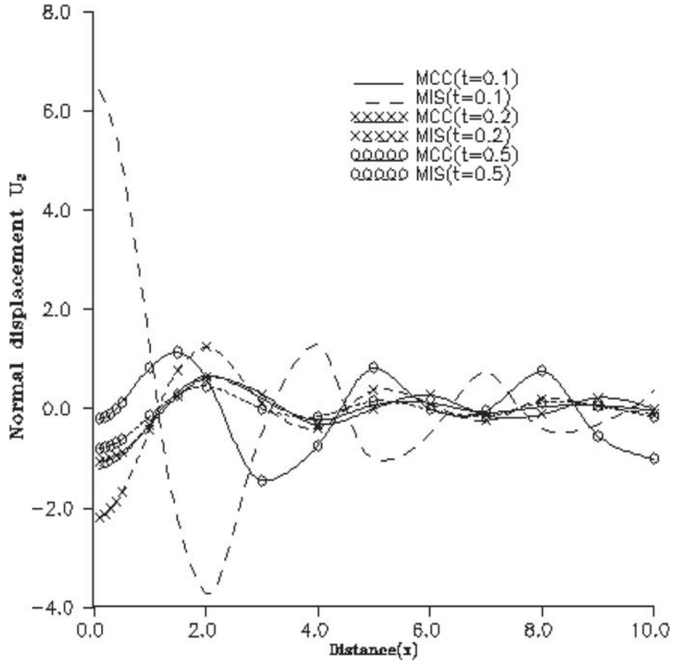

Fig. 1. Variation of normal displacement $U_2(=u_2/F)$ with distance x for concentrated force.

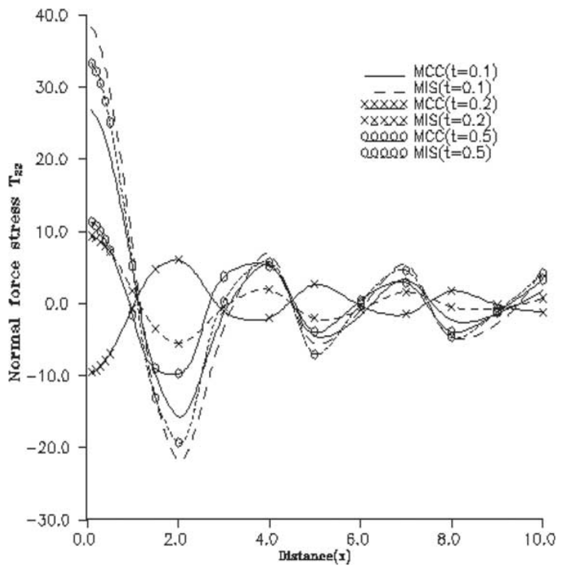

Fig. 2. Variation of normal force stress $T_{22}(=t_{22}/F)$ with distance x for concentrated force.

$$\rho = 2.19\ \text{gm/cm}^3, \quad \lambda = 7.59 \times 10^{10}\ \text{dyne/cm}^2, \quad \mu = 1.89 \times 10^{10}\ \text{dyne/cm}^2,$$
$$K = 0.0149 \times 10^{10}\ \text{dyne/cm}^2, \quad \gamma = 0.0268 \times 10^{10}\ \text{dyne}, \quad j = 0.00196\ \text{cm}^2.$$

The values of normal displacement $U_2 = (u_2/F)$ normal force stress $T_{22} = (t_{22}/F)$ and tangential couple stress $M_{23} = (m_{23}/F)$ for a micropolar cubic crystal (MCC) and micropolar isotropic solid (MIS) have been studied at $t = 0.1$, 0.2 and 0.5 and the variations of these components with distance x have been shown by (a) solid line (——) for MCC and dashed line (- - - -) for MIS at t=0.1, (b) solid line with centered symbol (×—×—×) for MCC and dashed line with centered symbol (×---×---×) for MIS at $t = 0.2$ and (c) solid line

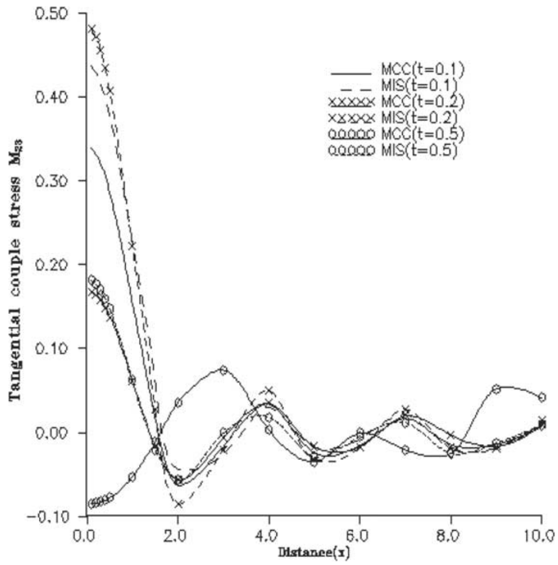

Fig. 3. Variation of tangential couple stress $M_{23}(=m_{23}/F)$ with distance x for concentrated force.

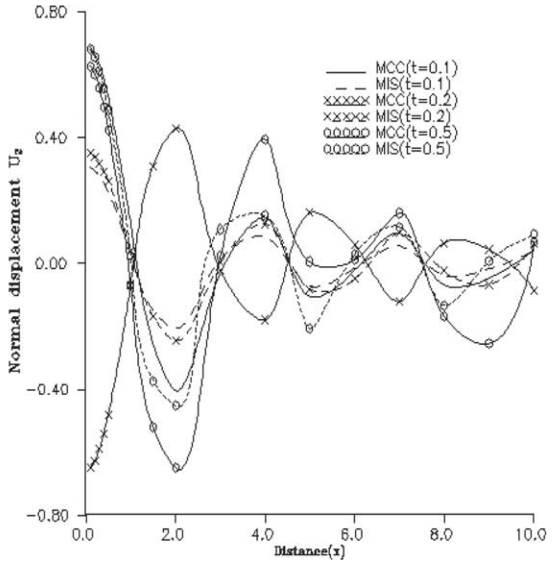

Fig. 4. Variation of normal force stress $U_2(=u_2/F)$ with distance x for uniformly distributed force.

with centered symbol (○—○—○) for MCC and dashed line with centered symbol (○- - -○- - -○) for MIS at $t = 0.5$. These variations are shown in Figs. 1–9. The comparison between micropolar cubic crystal and micropolar isotropic solid is shown. The computations are carried out for $y = 1.0$ in the range $0 \leqslant x \leqslant 10.0$.

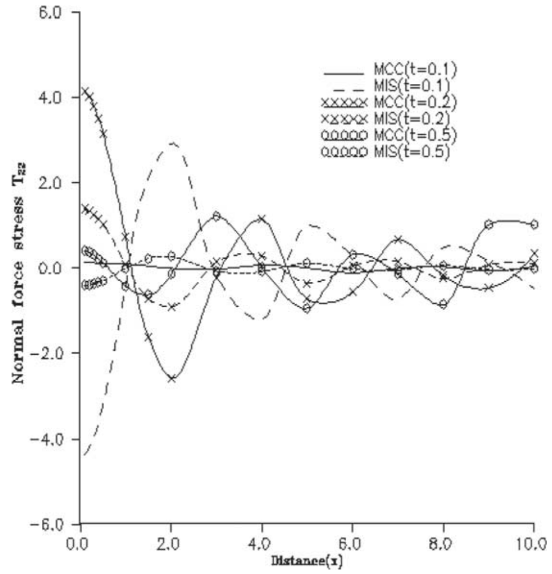

Fig. 5. Variation of normal force stress $T_{22}(=t_{22}/F)$ with distance x for uniformly distributed force.

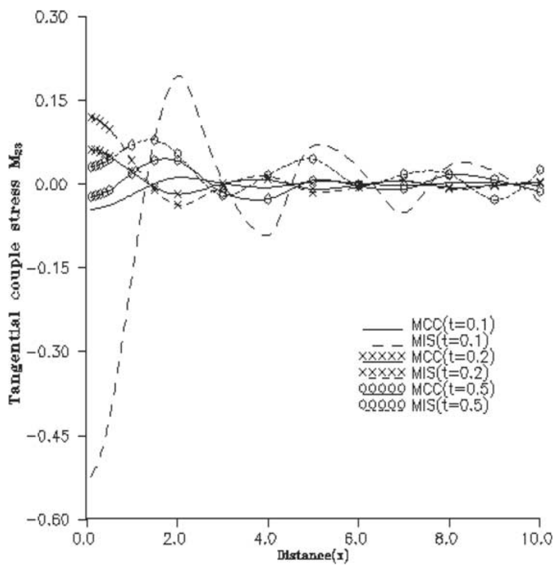

Fig. 6. Variation of tangential couple stress $M_{23}(=m_{23}/F)$ with distance x for uniformly distributed force.

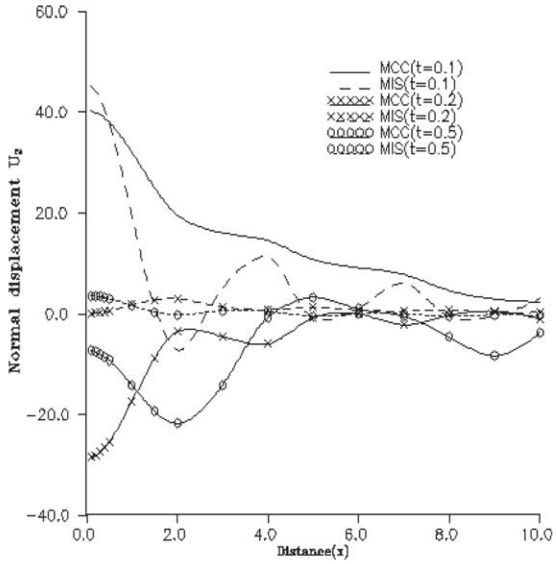

Fig. 7. Variation of normal displacement $U_2(=u_2/F)$ with distance x for linearly distributed force.

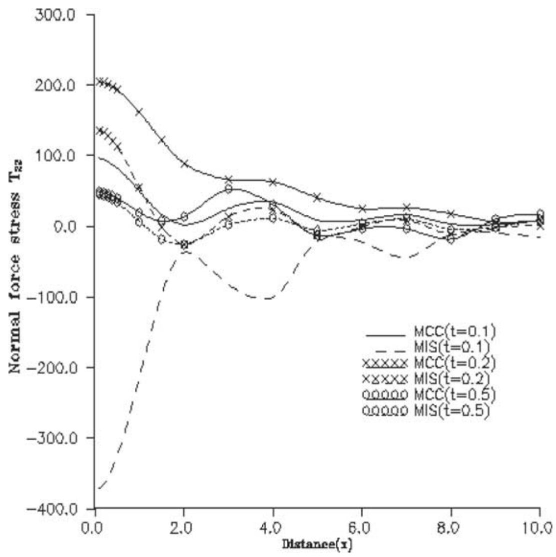

Fig. 8. Variation of normal force stress $T_{22}(=t_{22}/F)$ with distance x for linearly distributed force.

6. Discussions for various cases

6.1. Concentrated force

The variations of normal displacement, normal force stress and tangential couple stress being oscillatory are similar in nature with difference in their magnitudes. Although the values of normal force stress are more as compared to normal displacement and tangential couple stress, it is observed that the magnitude

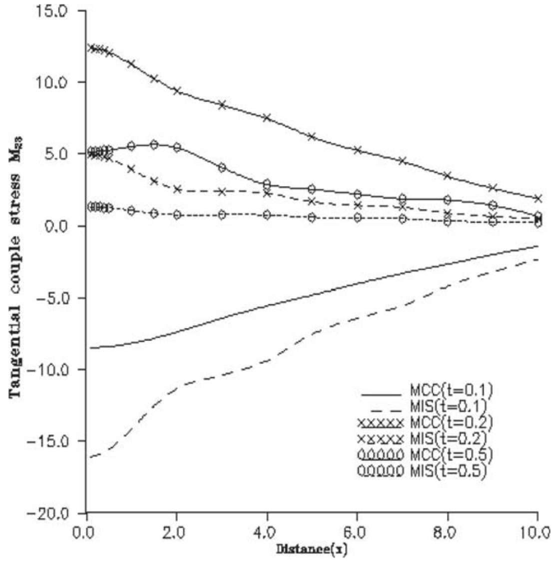

Fig. 9. Variation of tangential couple stress $M_{23}(=m_{23}/F)$ with distance x for linearly distributed force.

of oscillations decrease with increase in time. Also the values of all the quantities decrease with increase in horizontal distance. The variations of normal displacement, normal force stress and tangential couple stress are shown in Figs. 1–3 respectively.

6.2. Uniformly distributed force

The variations of all the quantities are similar in nature to the variations obtained in case of concentrated force. However the values of all the quantities for MIS are large as compared to the values for MCC and hence to compare the variations among both the solids, the values of normal displacement, normal force stress and tangential couple stress for MIS have been demagnified by 10, 100 and 10 respectively. The variations of normal displacement, normal force stress and tangential couple stress in case of uniformly distributed force are shown in Figs. 4–6 respectively.

6.3. Linearly distributed force

It is observed that the oscillations of the variations of quantities are less as compared to the oscillations obtained on the application of concentrated force and uniformly distributed force. Also, the values of normal force stress and tangential couple stress, very close to the point of application of source, are more for MCC as compared to MIS but the variation for normal displacement are opposite in nature at the same point. The variations of normal displacement, normal force stress and tangential couple stress shown in Figs. 7–9 respectively depicts that the variations of all the quantities converges to zero with increase in horizontal distance.

7. Conclusion

The properties of a body depend largely on the direction of symmetry. The values of all the quantities decrease with increase in time for various forces. The values of all these quantities for MIS are large as compared to MCC when uniformly distributed force is applied. The values of normal displacement, normal force stress and tangential couple stress are less when concentrated force is applied or we may say that the body is deformed to a much more extent on the application of strip loading.

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